The velocity field obtained can be used for analysis of an non-steady-state boundary layer at the porous surface of a moving body.

LITERATURE CITED

1. A. S. Gupta, "Ekman layers on a porous plate," Phys. Fluids, 5, Part 5, 930-931 (1972).
2. A. A. Gurchenko and V. M. Rogovoi, "The not fully established boundary layer at a rotating plate," in: Dynamics of Elastic Construction with a Liquid [in Russian], Tomsk (1977).
3. A. M. Éfros and A. M. Danilevskii, Operational Calculation and Contour Integrals [in Russian], Kharkov (1937).

COMPLEX HEAT EXCHANGE OF A DISPERSED TURBULENT FLOW IN A PIPE

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UDC 536.2–3

1. We will write the energy equation for a turbulent flow of a gas and suspended matter in a pipe in the form [1, 2]

$$(1-\beta)c\rho u\frac{\partial T}{\partial x}+\beta c_1\rho_1 u_1\frac{\partial T_1}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left\{r(\lambda+\lambda_t)\frac{\partial T}{\partial r}-r\beta c_1\rho_1\langle v_1' T_1'\rangle\right\}+\operatorname{div}\mathbf{q}_r, \tag{1.1}$$

where β is the volume concentration of the solid phase, λ and λ_t are the molecular and turbulent thermal conductivity, respectively, $\langle v_1' T_1'\rangle$ is the turbulent energy transport by particles, and $\mathbf{q}_r$ is the resultant radiation flux.

It is necessary for a description of the heat exchange of a two-phase flow to supplement Eq. (1.1) with an energy equation for the particles. We will discuss the turbulent flow of a gas and suspended matter with a low concentration of heavy particles, i.e., $\beta \ll 1$. The particles are distributed uniformly over the cross section of the pipe. Then Eq. (1.1), upon neglect of terms containing β, is written in dimensionless form as

$$\frac{u}{\bar u}\frac{\partial\Theta}{\partial\bar x}=\frac{4}{\mathrm{Re}}\frac{1}{\bar r}\frac{\partial}{\partial\bar r}\left\{\bar r\left(\frac{1}{\mathrm{Pr}}+\frac{1}{\mathrm{Pr}_t}\frac{\nu_t}{\nu}\right)\frac{\partial\Theta}{\partial\bar r}\right\}+\frac{2}{\mathrm{Bo}}\operatorname{div}\bar{\mathbf{q}}_r, \tag{1.2}$$

where $\Theta = T/T_0$, $\bar x = x/D$, $\bar r = r/R$, $\mathrm{Bo} = c\rho\bar u/\sigma_0 T_0^3$, $\mathrm{Re} = \bar u D/\nu$; $\bar u$ is the average velocity, Pr_t is the turbulent Prandtl number, T_0 is the ambient temperature at the pipe entrance ($T_0 > T_1$), and ν_t is the turbulent viscosity. The boundary conditions for (1.2) are of the form

$$\Theta(\bar r;\,0)=1,\ \Theta(\bar x>0;\,1)=\Theta_1.$$

2. The velocity distribution in viscous and transition layers is given in [3] in the form

$$\frac{u}{v^*}=\frac{2.3}{\varkappa_0}\lg\frac{v^*y}{\nu}+5.8,\quad 30\leqslant\frac{v^*y}{\nu}\leqslant 700;$$

$u/v^* = v^*y/\nu$, and $v^*y/\nu < 30$ (y is the distance from the wall).

The value of the tangential stress τ in a dispersed flow is related to the analogous quantity in a pure gas flow τ_0 by the relationship

$$\tau/\tau_0 = 1+\eta\mu, \tag{2.1}$$

where μ is the discharge concentration and η is a coefficient reflecting the strength of the effect of μ on the degree of deformation of the velocity distribution in a two-phase flow.

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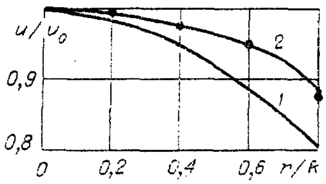

Fig. 1

Using von Karman's turbulent drag law and Eq. (2.1), one can show approximately that

$$\frac{\tau}{\tau_0} \simeq \frac{\varkappa^2}{\varkappa_0^2} k, \tag{2.2}$$

where the subscript "0" refers to quantities of a pure flow and k is an empirical coefficient. It follows from (2.2) that the constant in the drag law for a two-phase flow is related to von Karman's constant by the relationship

$$\varkappa^2 \simeq \frac{\varkappa_0^2 (1 + \eta\mu)}{k}.$$

We use the von Karman distribution [4] for the description of the velocity in the core of the flow of the carrier medium which with (2.1) and (2.2) taken into account has the form

$$\frac{u_0 - u}{v^*} = -\frac{k}{\varkappa_0 \sqrt{1 + \eta\mu}} \left[\ln\left(1 - \sqrt{1 - \frac{y}{R}}\right) + \sqrt{1 - \frac{y}{R}}\right], \qquad \frac{v^*}{\nu} y > 700, \tag{2.3}$$

for a pipe, where u_0 is the velocity on the pipe axis. With k = 1 and $\mu = 0$ Eq. (2.3) changes into the formula for the velocity distribution for a dust-free gas [4]. The velocity distributions in the core of the flow with an average velocity of 25.1 m/sec in the pipe are given in Fig. 1 as calculated from Eq. (2.3). Curve 1 corresponds to a particle-free gas ($\mu = 0$), and curve 2 corresponds to a two-phase flow with a particle concentration $\mu = 2.86$ kg/kg. The value k = 1 was adopted in the calculations. The value of the coefficient η in Eq. (2.1) was taken from the data of [3]. The experimental results of [3] are denoted by filled circles. The value of the turbulent viscosity in (1.2) should take account of the effect of particles on the turbulent structure of the carrier medium. We will adopt the model of [5, 6] to describe the turbulent transport.

According to this model, one can represent the value of the turbulent viscosity within the confines of viscous and transition layers in the form

$$\nu_T(y) = A_0(y)\psi^2(y), \tag{2.4}$$

where

$$A_0(y) \simeq \varkappa_0^2 y^2 \left|\frac{\partial \bar{u}}{\partial y}\right|; \quad \psi(y) = \int_0^{t_m(y)} R(t)\,dt \Big/ \int_0^\infty R(t)\,dt;$$

t is the time, $\varkappa_0$ is the von Karman constant, $R(t)$ is the Lagrangian correlation coefficient of the transverse pulsation component of the velocity, and $t_m(y)$ is the limiting scale of the turbulent eddies such that eddies with scales $t > t_m(y)$ cannot approach the wall to within a distance less than y. As follows from [7, 8] the effect of the presence of particles is significant in the region of small wave numbers. One can assume for the sake of simplification that the effect of particles reduces to a cutoff of the short-wavelength part of the spectrum [6].

In the case of the Lagrangian turbulence description a cutoff of the spectrum in the region of large wave numbers is equivalent to the replacement of R(t) for a one-phase medium by the function [6]

$$R'(t) = R(t)Y(t - t'), \tag{2.5}$$

where $Y(x) = \begin{cases} 1, & x>0 \\ 0, & x<0 \end{cases}$, $t' = c'y_0/u$, and $y_0 = 1/k_0$ is some characteristic scale proportional to the particle diameter d which increases as μ increases in the region of small μ.

Substituting (2.5) into (2.4) and carrying out the necessary operations, we obtain for a pipe

$$\frac{\nu_T}{\nu} = \varkappa^2 (1-\bar{r})^2 \left[1 - \exp\frac{r - r_0}{\alpha_1}\right]^2 \left|\frac{\partial u}{\partial \bar{r}}\right|,$$

and $\alpha_1 = \alpha\nu/Rv^*$, where $\alpha = 30.4$ [6]. We will determine the value of y_0 from the notions that it should be proportional to the particle diameter d and increase as μ increases:

$$y_0 = a\mu d \quad \text{or} \quad \bar{r}_0 = 1 - a\mu d/R,$$

where a is a proportionality coefficient.

3. The authors of [9] and [10] obtained the value of $\operatorname{div}\bar{\mathbf{q}}_r$ in the energy equations from the solution of the transfer equation of radiation energy in the P_1-approximation of the method of spherical harmonics. It was assumed that the radiation is gray and irradiates only the particles. One can calculate the absorption and scattering coefficients and the average scattering cosine for a layer of particles on the basis of the theory of G. Mie if the complex refractive index of the particle material, the particle size distribution function, and the particle concentration are known. The expression for $\operatorname{div}\bar{\mathbf{q}}_r$ is obtained in the form

$$\operatorname{div}\bar{\mathbf{q}}_r = -4(1-\gamma)\tau_0 \left\{A_1 I_0(\sigma\bar{r}) - \Theta^4(\bar{r}) - I_0(\sigma\bar{r}) \int_0^{\sigma\bar{r}} \Theta^4(\bar{r}')\sigma\bar{r}' K_0(\sigma\bar{r}')\, d\sigma\bar{r}' + K_0(\sigma\bar{r}) \int_0^{\sigma\bar{r}} \Theta^4(\bar{r}')\,\sigma\bar{r}' I_0(\sigma\bar{r}')\, d\sigma\bar{r}'\right\},$$

where τ_0 is the optical thickness, $\sigma = \tau_0\sqrt{3(1-\gamma)(1-\gamma\bar{\mu})}$; γ is the ratio of the average scattering coefficient to the absorption coefficient, I_0 is the modified Bessel function, A_1 is a constant determinable from the boundary conditions for the transfer equation, and $\bar{\mu}$ is the average cosine of the scattering ($\bar{\mu} = 0$ for a spherical indicatrix). The value of the optical thickness τ_0 is directly proportional to the particle concentration, namely, $\tau_0 = 0.25\mu\rho R\langle k\rangle$, where $\langle k\rangle = \int_0^\infty B(\lambda; T) \int_0^\infty k \frac{dS}{dl}\, dl\, d\lambda \Big/ \int_0^\infty B(\lambda; T)\, d\lambda$ is the average absorption coefficient, k is the absorption coefficient of a particle, dS/dl is the distribution function of the specific surface of the particles with respect to their sizes l, $B(\lambda, T)$ is the Planck function, and λ is the wavelength.

We will not discuss particles of any particular material in this paper, and we adopt $\tau_0 = 2\mu$, thereby assuming that $(1/4)\rho R\langle k\rangle = 2$.

4. Equation (1.2) was solved numerically with the use of an implicit difference scheme solved by elimination. The algorithm of the problem was run on a BÉSM-6 computer. It is possible to calculate from the computed temperature field in the pipe the values of the Nusselt number of a convective heat flux

$$\mathrm{Nu}_c = 2\frac{\partial\Theta}{\partial\bar{r}}\Big/(\bar{\Theta} - \Theta_1) \qquad (\bar{r} = 1)$$

and the criterion Nu_Σ of the total heat flux

$$\mathrm{Nu}_\Sigma = \mathrm{Nu}_c + \mathrm{Nu}_r = \frac{2\frac{\partial\Theta}{\partial\bar{r}} + \frac{\mathrm{Pe}}{\mathrm{Bo}}\bar{q}_r}{\bar{\Theta} - \Theta_1} \qquad (\bar{r} = 1),$$

where

$$\bar{\Theta} = \int_0^1 \frac{u}{\bar{u}}\Theta\bar{r}\, d\bar{r} \Big/ \int_0^1 \frac{u}{\bar{u}}\bar{r}\, d\bar{r}.$$

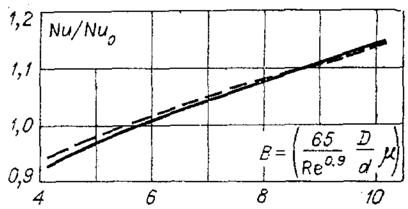

Fig. 2

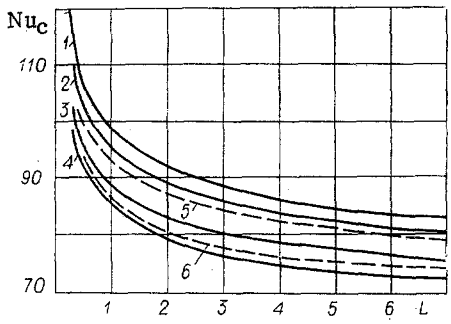

Fig. 3

The problem of purely convective heat exchange of a dust—air flow was solved first with various Re $\geqslant$ 2000 and $0 < \mu < 6$. The drag law [10] $\xi = \xi_0(1 + 0.145\,\mu)$, where ξ_0 is the Blasius law, was used in the calculations of the dynamic velocity v*. The computational results were worked up in the form of a dependence of $Nu(\bar{x} = 70)/Nu_0(\bar{x} = 70)$ on the parameter combination $B = (65/Re^{0.9})D\mu/d$. The quantity Nu_0 was calculated from the formula

$$Nu_0 = 0.022\,Re^{0.8}Pr^{0.4}.$$

Calculations by the numerical method are shown as a dashed curve in Fig. 2. The solid curve is plotted from the relationship

$$\frac{Nu}{Nu_0} = 0.64\left(\frac{65}{Re^{0.9}}\frac{D}{d}\mu\right)^{0.25},$$

which is a generalization of the experimental data of different authors and is derived in [12]. The computational results are in good agreement with the experimental values for small values of μ. This agreement is obtained with $Pr_t = 1$, $a = 1$, and $k = 1$. The problem of complex heat exchange was solved with Re = 30,000, Bo = 30, Pr = 0.7, $Pr_t = 1$, $a = 1$, $\gamma = 0.8$, $\bar{\mu} = 0.8$, and $\varepsilon = 1$.

The dependences of the value of the Nusselt criterion on channel length for local convective heat flow with complex heat exchange are given in Fig. 3 for various particle concentrations (optical thickness) (Re = $3\cdot10^5$, Bo = 30, Pr = 0.7, $Pr_t = 1$, $\gamma = 0.8$, $\bar{\mu} = 0.8$, $\varepsilon = 1$, and d/D = 0.0025; $\mu = 3$ for curves 1 and 5, $\mu = 2$ for curve 2, $\mu = 1$ for curves 3 and 6, and $\mu = 0.5$ for curve 4). The results for nonemitting particles are denoted by the dashed curves. In this case of complex heat exchange the values of the convective heat transfer differ from the heat transfer for purely convective heat exchange, other conditions being equal. At high wall temperatures the difference reaches 20%.

The temperature dependence of the local dimensionless heat flux transmitted by radiation and convection with $\mu = 0.5$, $\bar{q}_\Sigma = (q_c + q_r)/\bar{u}c\rho T_0$, is given in Fig. 4 (dashed curves) (the parameters are the same as in Fig. 3; L = 3.55 for curve 1, and L = 6.05 for curve 2) for various distances from the entrance. The analogous dependence for a local dimensionless heat flow by radiation (solid curves) $\bar{q}_r = q_r/\bar{u}c\rho T_0$ is also given there.

The dependences of the local Nusselt number for a radiative flux $Nu_r = \frac{Pe}{Bo}\bar{q}_r/(\bar{\Theta} - \Theta_1)$ on the particle concentration are given in Fig. 5 (the parameters are the same as in Figs. 3 and

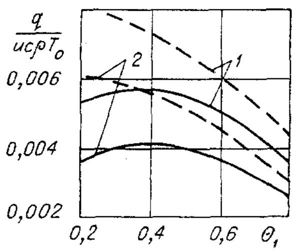

Fig. 4

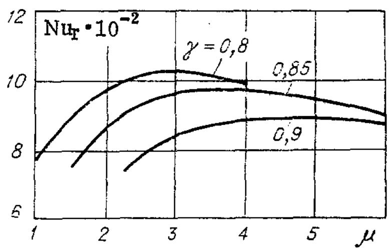

Fig. 5

4; $\Theta_1 = 0.8$, and $L = 24$) for various values of γ. The extremal behavior of the curves as μ increases is explained by the fact that an increase in the particle concentration (optical thickness) first leads to an increase in the heat transfer by radiation, since the cold layers adjacent to the channel walls of the moving medium absorb the radiation of the core of the flow weakly due to the optical thickness. Upon a further increase in the particle concentration the boundary layers shield the radiation of the flow core more and more strongly, and the heat exchange, having reached a maximum, decreases.

LITERATURE CITED

1. C. Coy, Hydrodynamics of Multiphase Systems [Russian translation], Mir, Moscow (1971).
2. Z. R. Gorbis, Heat Exchange and Hydromechanics of Dispersed Open Flows [in Russian], Énergiya, Moscow (1970).
3. N. I. Gel'perin, V. G. Ainshtein, L. I. Krupnik, and Z. N. Mamedlyaev, "Hydrodynamic drag of flows of gas and suspended matter," Izv. Vyssh. Uchebn. Zaved., Energ., No. 2 (1976).
4. G. Schlichting, Boundary-Layer Theory [Russian translation], Nauka, Moscow (1969).
5. J. A. Bujewitsch, (Yu. A. Buevich) "Comment on the construction of models for turbulence near a wall" [in German]," Z. Angew. Math. Mech., 49, 372 (1969).
6. Yu. A. Buevich, "A model for reducing the drag upon the introduction of particles into a turbulent flow of a viscous fluid," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 2, (1970).
7. Yu. A. Buevich and Yu. P. Gupalo, "The effect of suspended particles on the degeneration of isotropic turbulence," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1965).
8. Yu. A. Buevich and Yu. P. Gupalo, "Distortion of the energy spectrum of degenerating isotropic turbulence under the action of suspended particles," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1965).
9. K. M. Case and P. F. Zweifel, Linear Transport Theory, Addison-Wesley (1967).
10. M. N. Otsisik, Complex Heat Exchange [Russian translation], Mir, Moscow (1976).
11. F. F. Tsvetkov, "An investigation of local heat transfer from the wall of a pipe to a turbulent gas flow carrying suspended solid particles," Candidate's Dissertation, MÉI, Moscow (1967).
12. A. S. Sukomel, F. F. Tsvetkov, and R. R. Kerimov, Heat Exchange and Hydrodynamic Drag in Connection with the Motion of a Gas and Suspended Matter in Pipes [in Russian], Énergiya, Moscow (1977).

EFFECT OF INHOMOGENEITY OF THE DISPERSE PHASE ON COALESCENCE AND MASS-TRANSFER PROCESSES IN LIQUID EMULSIONS

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The coalescence of drops of a disperse phase serves as the basis for the separation of immiscible liquids in the absence of extraction and mass-transfer processes, in the chemical, pharmaceutical, food, and many other branches of industry [1]. In recent years great progress has been made in the use of highly effective demulsifiers to break up petroleum emulsions in industrial plants and oil refineries [2, 3]. However, interaction between drops, which has an exceptionally great effect on mass transfer and chemical reactions in the disperse phase, has been insufficiently studied [4].

The coalescence of drops under the action of agitation, in particular with the motion of an emulsion under turbulent conditions, is bound up with an increase in the rate of a broad range of technological processes and with an increase in the quality of mass-transfer and coagulation processes in pipelines and apparatus. An analysis of the interaction between finely dispersed drops with the break-up of emulsions by chemical methods using demulsifiers also makes it possible to solve the problem of two-phase flows in pipelines; these methods

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